DECOMPOSITION BASED PARALLEL HYBRID MOEA WITH APPLICATION TO THE MULTIOBJECTIVE MULTIDIMENSIONAL KNAPSACK PROBLEM

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ABSTRACT

Recently, there has been a noticeable tendency in research for combinatorial optimization issues toward the hybridization of metaheuristics with other optimization techniques. On the other hand, parallel conception of multiobjective evolutionary algorithms (MOEAs) provides a significant enhancements in terms of efficiency and effectiveness. In this paper, we propose a hybrid parallel multiobjective evolutionary algorithm, with an application to the multiobjective multidimensional Knapsack Problem (MOMKP). The suggested approach can be considered as an enhanced parallel variant of two-phase method. Finally, we present an experimental study, where we assess the suggested approach against state-of-the-art sequential and parallel MOEAs, as to emphasize the contribution of the search strategy of the parallel MOEAs and its ability to approximate target areas of the true Pareto Front.

1. INTRODUCTION

Multiobjective Problems consists to optimize k objective functions simultaneously. The general form of MOPs is stated as follows :

$$\begin{cases} "max" Z(x) = (Z^1(x), Z^2(x), \ldots, Z^k(x)), \\ s.t., \quad x \in \Omega. \end{cases}$$

where Ω is the decision space, $x \in \Omega$ is a decision vector, and the vector $Z(x)$ consists of k objective functions $Z^i(x) : \Omega \to \mathbb{D}_i$, $i \in \{1,...,k\}$. Since the aim in MOPs is to find good compromises. Here, we present the dominance relation, as to define optimality in MOPs. For any couple of feasible solutions x and x' in Ω, the vector $Z(x) = (Z^1(x),...,Z^k(x))$ is said to dominate the vector $Z(x') = (Z^1(x'), \ldots, Z^k(x'))$, denoted as $Z(x) \succ Z(x')$, if and only if, $\forall i \in \{1,\ldots,k\}$, $Z^i(x) \leq Z^i(x')$ and $Z(x) \neq Z(x')$. A feasible solution $x^* \in \Omega$ is called a Pareto optimal solution or an efficient solution, if and only if, $\nexists y \in \Omega$ such that $Z(y) \succ Z(x^*)$. The set of Pareto optimal solutions is called the Pareto-optimal Set (PS) : $PS = \{x \in \Omega | \ \nexists y \in \Omega, Z(y) \succ Z(x^*)\}$. The evaluation of solutions in PS is called the Pareto Front (PF) : $PF = \{Z(x) | x \in PS\}$.

Furthermore, there exists an important classification of efficient solutions : supported efficient solutions and non-supported efficient solutions. According to Geoffrion's theorem [8], the supported efficient solutions, denoted X_{SE}, can be obtained by solving the parametric single-objective problems obtained by a linear aggregation of the different objectives P_λ :

$$(P_\lambda) \begin{cases} max \sum_{i=1}^{k} \lambda_i Z^i(x), \\ s.t., \quad x \in \Omega, \end{cases}$$

where, $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_k) \in \mathbb{R}_+^k$ is a weight vector with all positive components.

On the other hand, there exists the non-supported efficient solutions set, denoted X_{NS}, this subset of the efficient solutions set cannot be obtained by solving P_λ. Furthermore, the images of the unsupported solutions are not located on the boundary of the convex envelope.

In this paper, we propose a parallel hybrid multiobjective evolutionary algorithm, designed in a master/salve model, we call it Decomposition Based Parallel Hybrid MOEA (D/PHMOEA). The suggested algorithm is an enhanced two-phase type algorithm, where the first phase consists of finding the supported solutions set using an exact method. In the second phase, the decision space is structurally decomposed and allocated to multiple MOEAs operating in parallel. Each MOEA is dedicated to a specific region of the decision space that is initially characterized by a subset of the supported solutions found in the first phase. We assess the suggested method against some successful MOEAs using benchmark instances of the Multiobjective Multidimensional Knapsack Problem (MOMKP). This latter is a variant of the Knapsack Problem (KP), which is known to be NP-hard [6]. Mathematically, MOMKP can be stated as follows : given n items having p characteristics (weight, volume, etc.) $w_j^i \geq 0$, where, $j \in \{1,\ldots,p\}$, $i \in \{1,\ldots,n\}$, and m profits c_j^k, $k \in \{1,\ldots,m\}$, we want to select items as to maximize the m total profits, while not exceeding the p knapsack capacities W_i with regards to the different characteristics. The MOMKP is formulated as follows :

$$(MOMKP)\begin{cases} \text{"}max\text{"}\ Z^k(x) = \sum_{j=1}^{n} c_j^k x_j, \quad k \in \{1,\ldots,m\} \\ s.t., \quad \sum_{j=1}^{n} w_j^i x_j \leq W_i, \quad j \in \{1,\ldots,p\} \\ x_j \in \{0,1\}, \forall j \in \{1,\ldots,n\}. \end{cases}$$

2. SUGGESTED ALGORITHM (D/PHMOEA)

2.1. Description

In this section, we present a resumed description of the suggested algorithm, which is, as we already mentioned, an enhanced variant of the two-phase method. The first phase of the suggested algorithm method remains unchanged, as it is the case for all two-phase algorithms. It consists in the construction of the set of efficient solutions supported by the dichotomy method proposed by Aneja & Nair [9], based on Geoffrion's theorem [8]. This algorithm generates all the supported efficient solutions, including extreme and non-extreme ones, using a single objective problem whose objective function is a linear aggregation of two objectives (see P_λ in the introduction). Next, after having the set of supported efficient solutions in hand, the second phase consists of approximating the set of non-supported solutions using multiple asynchronous parallel MOEAs. Each one of the parallel search entity is designed to target a specific region of the Pareto optimal front. This is by initializing its archive solutions set using a subset of the supported efficient solutions set gathered form the same region. Furthermore, the selection operator is defined according to the following order relation : let P_t be the current population of a search entity, PS_t be the set of Pareto solutions obtained at iteration t (i.e., $= \{x \in P_t | \ \nexists y \in P_t : y \succ x\}$), and $R \subset Z(PS_t) \cap Z(X_{SE})$ the extreme points enclosing the predefined region for the search entity, $|R| = k$ the number of objective functions. The order relation is defined as follows :

$$\forall x, y \in P_t, x \geq y \iff (x \succeq y) \vee (\phi(x) \geq \phi(y)),$$

where,

$$\phi(x) = \sum_{i=1}^{k} \left(Z^i(x) \sum_{j=1}^{k} \frac{R_j^i}{||\sum_{j=1}^{k} R_j||_2} \right).$$

Hence, the process of selecting individuals that pass to the next generation P_{t+1} is given explicitly as follows :

$$P_{t+1} = \{x \in P_t | (x \in PS_t) \vee (rank(x, P_t \backslash PS_t \leq N))\},$$

where, $rank(x,P)$ is the order of a solution x compared to elements of a set P according to the function ϕ, and N is the parameter fixing the size of the current directing population.

The suggested pMOEA can be classified as a cooperative algorithmic level parallel model designed in a master/worker paradigm, handling : (1) a master entity in charge of gathering and computing the global approximated Pareto solutions, (2) multiple MOEAs with directed the search to specific regions of the true Pareto front, with the help of a specific selection operator described above defined with a subset of supported efficient solutions. Regarding the decomposition procedure, this occurs over the decision space using the supported efficient solutions set found in the first phase. This is by partitioning this set into p equally sized sets, according to one of the objective functions. As we mentioned earlier, the extreme solutions of each subset is used to construct the selection operator of each parallel MOEAs.

Figure 1 presents an example of the decomposition procedure applied to a bi-objective Knapsack instance : 2KP100-TA-0 [10]. The decision space is decomposed into $p = 4$ sub-regions.

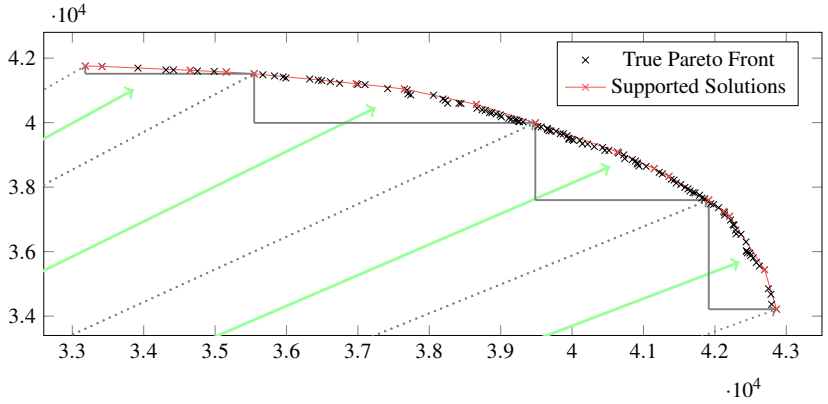

FIGURE 1 – Illustrative example of the used decision space decomposition (Bazgan KP instance [10], 2KP100-TA-0)

2.2. Experimental results

We tested the sugessted algorithm on benchmark instances of MOMKP chosen from the instance libraries : Zitzler and al. [7], of which we consider for this experiments three instances with the number of items 250, 500, and 750, with two objective functions. We compared the performance of the suggested algorithm three four multiobjective algorithms with different concepts and/or different search strategies : NSGAII [2], SPEA2 [3], MOEA/D [4], MOFPA [12], PCP-MOEA [11]. The evaluation and comparison of the obtained solution's the quality, one must consider (convergence, and the spread), we used three performance metrics : Inverted Generational Distance (IGD) [13], Hypervolume [7], and the set coverage metric [13].

Table 1 resumes the obtained values of the IGD metric assessing the convergence of the obtained Pareto sets. The IGD values shows clearly that, in general, D/PHMOEA converges better than all of the competing algorithms, especially for large instances, with the exception of the instance 2.250.

Instance	Algorithm				
	SPEA2	MOEA/D	MOFPA	PCPMOEA	D/PHMOEA
250	14.883	3.9690	0.7248	**0.2964**	0.5248
500	79.743	14.466	2.2850	0.7961	**0.3226**
750	224.794	32.655	10.062	3.1302	**2.1359**

TABLE 1 – Experimental results concerning the IGD metric of the MOMKP instances.

Table 2 resumes the obtained results regarding the Hypervolume indicator. This indicator is used to evaluate the convergence to the true Pareto front and diversity of the obtained Pareto front. As it is shown below, it is obvious that the suggested algorithm produces higher quality fronts, with significant difference especially when compared to SPEA2 and MOEA/D, and it's at least comparable to MOFPA and PCPMOEA algorithms.

Instance	Algorithm				
	SPEA2	MOEA/D	MOFPA	PCPMOEA	D/PHMOEA
250	9.1677527E+7	9.8374725E+7	9.8556257E+7	9.8654313e+7	**9.8692999E+7**
500	3.6944050E+8	4.0515241E+8	4.0707772E+8	4.0772607E+8	**4.0787113E+8**
750	7.8038570E+8	8.8553814E+8	8.8572075E+8	8.9260224E+8	**8.9351766e+8**

TABLE 2 – Experimental results concerning the Hypervolume indicator of the MOMKP instances.

Table 3 shows the obtained mean coverage values for each pair adduced as follows : the symboles $\succeq$ and $\preceq$ refer to $C(own\ algo.,\ competing\ algo.)$ and $C(competing\ algo.,\ own\ algo.)$ respectively. The results show that D/PHMOEA produces a better quality of Pareto fronts when compared to SPEA2, MOEA/D. However, PCPMOEA and MOFPA are shown to be the most competitive, especially for the smaller instances, although, the suggested algorithm maintained to be dominant, scoring an overall mean coverage values of 78% as dominant and 16% as dominated.

Instance		Algorithm			
		SPEA2	MOEA/D	MOFPA	PCPMOEA
250	$\succeq$	1	0.9930	0.6664	0.5783
	$\preceq$	0	0	0.2123	0.3772
500	$\succeq$	1	0.9995	0.8299	0.9177
	$\preceq$	0	0	0.0981	0.0047
750	$\succeq$	1	0.9985	0.8697	0.8531
	$\preceq$	0	0	0.0885	0.1122
Average	$\succeq$	1	0.9970	0.7886	0.7830
	$\preceq$	0	0	0.3989	0.1647

The last two rows contain the mean values for each column.

TABLE 3 – Coverage metric of the suggested algorithm against other competing algorithms.

Figure 2 presents an illustrative example of the obtained results. The visual observation confirms the fact that the suggested algorithm is at least comparable to recent state-of-the-art algorithms.

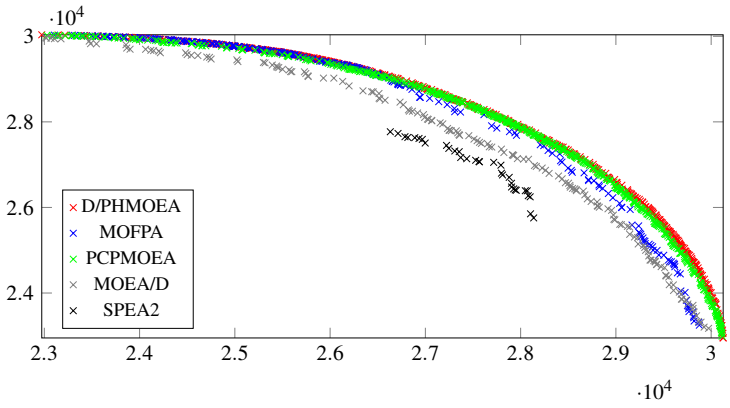

FIGURE 2 – Illustrative example of the obtained approximated Pareto fronts using SPEA2, MOEA/D, MOFPA, PCPMOEA, and D/PHMOEA.

3. CONCLUSIONS

In this paper, we presented a parallel two-phase type algorithm with an application to the multiobjective multidimensional Kanapsack Problem, called Decomposition based Parallel Hybrid MOEA (D/PHMOEA). The suggested algorithm is a hybrid algorithm, combining an exact method for finding the set of supported solutions, and a parallel MOEA with weighted-criteria selection operator, designed in a master/worker paradigm, as to target specific regions of the true Pareto set. The suggested algorithm has been assessed against state-of-the-art algorithms with different search strategies. The approach has shown conclusive results regarding the convergence and diversity of the evolved solutions.

4. REFERENCES

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ICMA 2021

7 – 8 Dec 2021, Blida, ALGERIA

Program

Time	7th Dec 2021		8th Dec 2021	
8:45	Opening Ceremony			
9:00				
9:15			Invited Talk 7	Invited Talk 8
9:30				
9:45	Invited Talk 1		Invited Talk 9	Invited Talk 10
10:00				
10:15	Coffee Break		Parallel Session S5	Parallel Session S6
10:30				
10:45	Invited Talk 2			
11:00				
11:15	Invited Talk 3			
11:30				
11:45	Invited Talk 4			
12:00				
12:15	Lunch		Lunch	Lunch
12:30				
12:45				
13:00				
13:15	Parallel Session S1	Parallel Sessions S2	Parallel Session S7	Parallel Session S8
13:30				
13:45				
14:00				
14:15				
14:30				
14:45				
15:00				
15:15	Invited Talk 5	Invited Talk 6	Invited Talk 11	Invited Talk 12
15:30				
15:45	Parallel Session S3	Parallel Session S4	Parallel Session S9	Parallel Session S10
16:00				
16:15				
16:30				
16:45				
17:00				
17:15				
17:30				
17:45			Closing Ceremony	
18:00				

ICMA 2021 7 – 8 Dec 2021, Blida, ALGERIA Invited talks		
Date and Time	**Speaker**	**Title**
T1. 7 Dec. 9.30 - 10.00 am	Prof. Taous Meriem Laleg, KAUST, KSA	Mathematical modeling of the arterial stiffness: solitons-based model versus fractional order model
T2. 7 Dec. 10.30 - 11.00 am	Prof. Frédéric Richard, Aix-Marseille Université, France	Anisotropic fractional Brownian fields: properties, simulation and estimation.
T3. 7 Dec. 11.00 - 11:30 am	Prof. Boualem Djehiche, KTH Royal Institute of Technology, Sweden	On a class of time-inconsistent optimal stopping problems
T4. 7 Dec. 11.30 - 12.00 am	Prof. Stefano De Marchi, University of Padova, Italy	Variably Scaled Discontinuous Kernels
T5. 7 Dec. 3.00 - 3.30 pm	Prof. Faouzia Rebbani, Ecole Supérieure de Technologies Industrielles ESTI Algeria	On some analytical and numerical aspects of some regularization methods applied to ill-posed problems in PDEs
T6. 7 Dec. 3.00 - 3.30 pm	Prof. Fatma Zohra Nouri, Badji Mokhtar - Annaba University, Algeria	Mathematical modeling of multiphase flows
T7. 8 Dec. 9.00 - 9.30 am	Prof. Mouffak Benchohra, Djillali Liabes University, Algeria	Semilinear Differential Equations: Existence, Stability and Controllability
T8. 8 Dec. 9.00 - 9.30 am	Prof. Samsul Ariffin B A Karim, Universiti Teknologi Petronas, Malaysia	Cubic B-Spline Approximation for Solving Linear Two-Point Boundary-Value Problems
T9. 8 Dec. 9.30 - 10.00 am	Prof. Chikh Bouzar, University of Oran, Algeria	Spaces of multi-anisotropic ultradifferentiable functions
T10. 8 Dec. 9.30 - 10.00 am	Prof. Amar Oukil, Sultan Qaboos university, Oman	Ranking frameworks based on the integration of Data Envelopment Analysis and Ordered Weighted Averaging
T11. 8 Dec. 3.00 - 3.30 pm	Prof. Dalila Azzam-Laouir, University of Jijel, Algeria	Differential inclusions governed by maximal monotone operators
T12. 8 Dec. 3.00 - 3.30 pm	Dr. Loqmane Seridi, Johnson & Johnson Co, USA	Gaining biological insights through mathematics

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Parallel Sessions

Date: 07/12/ 2021

Parallel Session S1		
13:00 – 13:15	Title:	Study of some non-autonomous abstract problems of elliptic type in an unbounded domain
	Presenter:	**Boutaous Fatiha**
13:15 – 13:30	Title:	Common Fixed Point for Multivalued (psi,theta,G)-Contraction Type Maps in Metric Spaces with a Graph Structure
	Presenter:	**Benchabane Saadia**
13:30 – 13:45	Title:	Existence, uniqueness and stability of solutions to a delay hematopoiesis model
	Presenter:	**Bouakkaz Ahlème**
13:45 – 14:00	Title:	Results in semi-E-convex functions
	Presenter:	**Ayache Benhadid**
14:00 – 14:15	Title:	A functional equation arising in dynamic programming via a generalized F- weak contractions of Hardy-Rogers.
	Presenter:	**Djamila Derouiche**
14:15 – 14:30	Title:	On the strongly mid p-summing operators and application
	Presenter:	**Ferradi Athmane**
14:30 – 14:45	Title:	A note on the influence of different additional regularity on the critical exponent
	Presenter:	**Khaldi Said**
14:45 – 15:00	Title:	Explicit limit cycle for class of multi-parameter polynomial differential system
	Presenter:	**Kina Abdelkrim**
Parallel Session S2		
13:00 – 13:15	Title:	Adaptive estimates for parafima models
	Presenter:	**Amimour Amine**
13:15 – 13:30	Title:	Periodic Negative Binomial INGARCH(1,1) Model
	Presenter:	**Abderrahmen Manaa**
13:30 – 13:45	**Title:**	On generalized integer-valued GARCHX model with structural changes
	Presenter:	**Mohamed Djemaa Sadoun**
13:45 – 14:00	Title:	Existence and uniqueness of solution to G-neutral stochastic differential equations
	Presenter:	**Zakaria Boumezbeur**
14:00 – 14:15	Title:	The almost complete convergence of the high-risk point kernel functional conditional estimate for quasi-associated data
	Presenter:	**Hamza Daoudi**

14:15 – 14:30	Title:	Multiplicative bias correction for inverse gamma and beta prime kernel density estimators
	Presenter:	**Harfouche Lynda**
14:30 – 14:45	Title:	Optimal Bandwidth selection in M-type estimate of the regression function in associated and left-truncated model
	Presenter:	**Asma Gheliem**
14:45 – 15:00	Title:	Probability Tail for Linearly Negative Quadrant Dependent Random Variables of Partial Sums and Application to Linear Model
	Presenter:	**Zoubeyr Kaddour**
Parallel Session S3		
15:30 – 15:45	Title:	Nontrivial solution for quasilinear elliptic systems in divergence form
	Presenter:	**Lecheheb Samira**
15:45 – 16:00	Title:	Multiple solutions for nonhomogeneous nonlocal elliptic problems with singular potentiale
	Presenter:	**Matallah Atika**
16:00 – 16:15	Title:	On nonhomogeneous p-laplacian elliptic equations involving a critical Sobolev exponent and multiple Hardy-type terms
	Presenter:	**Messirdi Sofiane**
16:15 – 16:30	Title:	First order evolution inclusions governed by sweeping process in banach spaces
	Presenter:	**Selamnia Fatiha**
16:30 – 16:45	Title:	Lower and upper solutions for conformable fractional differential equations
	Presenter:	**Bendouma Bouharket**
16:45 – 17:00	Title:	Stability analysis for a generalized proportional fractional langevin equation with variable coefficient and mixed integro–differential boundary conditions
	Presenter:	**Boutiara Abdellatif**
17:00 – 17:15	Title:	Some existence results to positive solutions for p-Laplacian boundary value problems of fractional differential equations
	Presenter:	**Chabane Fraid**
17:15 – 17:30	Title:	Nonlocal conditions for fractional differential equations
	Presenter:	**Dib Fatima**
17:30 – 17:45	Title:	Existence of unique solution of a fractional wave equation with free boundary conditions
	Presenter:	**Djemiat Rabah**
17:45 – 18:00	Title:	Existence of random coupled system of fractional differential equations in generalized Banach space with retarded and advanced arguments
	Presenter:	**Fredj Fouad**
Parallel Session S4		
15:30 – 15:45	Title:	i-packing and packing coloring of generalized Peterson graphs.
	Presenter:	**Daouya Laiche**
15:45 – 16:00	Title:	Graphs whose weak Roman domination number increases by the removal of any edge.
	Presenter:	**Rihab Hamid**

16:00 – 16:15	Title:	The effect of edge lifting on Roman domination in graphs.
	Presenter:	**Hicham Meraimi**
16:15 – 16:30	Title:	Power contamination and domination on the grid.
	Presenter:	**Amina Ainouche**
16:30 – 16:45	Title:	Strong Incidence Colouring of Graphs
	Presenter:	**Brahim Benmedjdoub**
16:45 – 17:00	Title:	Equitable coloring and Scheduling on identical machines
	Presenter:	**Sarah Nouri**
17:00 – 17:15	Title:	The m-machine chain-reentrant flow shop with two competing agents
	Presenter:	**Nazim Sami**
17:15 – 17:30	Title:	Two-machine flow shop scheduling problem with two competing agents
	Presenter:	**Abdennour Azerine**
17:30 – 17:45	Title:	Scheduling on batch processing machines with compatibility graphs
	Presenter:	**Khaoula Bouakaz**
17:45 – 18:00	Title:	Continuous global optimization using space-filling curve
	Presenter:	**Raouf Ziadi**
Date: 08/12/ 2021		
Parallel Session S5		
10:00 – 10:15	Title:	Uniqueness and stability of parameter identification in elliptic boundary value problem
	Presenter:	**Benyoucef Abir**
10:15 – 10:30	Title:	Optimal decay for abstract second-order evolution equation with infinite memory and time-varying delay in Hilbert spaces.
	Presenter:	**Chellaoua Houria**
10:30 – 10:45	Title:	Second-order differential inclusion with sum of two perturbations
	Presenter:	**Imen Boutana**
10:45 – 10:00	Title:	Anisotropic degenerate parabolic problems in RN with variable exponent and locally integrable data
	Presenter:	**Mecheter Rabah**
10:45 – 11:15	Title:	On the geometric-weighted-variable Hardy spaces on Lipschitz domains
	Presenter:	**Melkemi Oussama**
11:15 – 11:30	Title:	Some weak invariance results for fractional differential inclusion
	Presenter:	**Omar Benniche**
11:30 –	Title:	The lipschitz weakly p-nuclear operators and its injective hull

11:45	Presenter:	**Tiaiba Toufik**
11:45 – 12:00	Title:	Stability of an abstract system with infinite history
	Presenter:	**Youkana Abderrahmane**
Parallel Session S6 - Applied		
10:00 – 10:15	Title:	Nonlocal differential operators applied to image processing
	Presenter:	**Sabira Ben Alia**
10:15 – 10:30	Title:	Self-similar solutions for free-boundary problem from contour enhancement in image processing
	Presenter:	**Hossemddine Achour**
10:30 – 10:45	Title:	Stochastic differential equation in image restoration
	Presenter:	**Halilou Radhia**
10:45 – 10:00	Title:	TVD WAF scheme with PVRS Riemann Solver for the Drift-Flux Equations of Two-Phase Flows under Isothermal conditions
	Presenter:	**Souheyla Ouffa**
10:45 – 11:15	Title:	Hybrid conjugate gradient-BFGS methods based on Wolfe line search
	Presenter:	**Samia Khelladi**
11:15 – 11:30	Title:	Numerical simulation of a finite element bending support
	Presenter:	**abdelkader kirad**
11:30 – 11:45	Title:	The problem of the mixed boundary value of the elastic medium under torsion
	Presenter:	**Djamel Djamel**
11:45 – 12:00	Title:	SPDEs with space interactions and application to population modelling
	Presenter:	**Makhlouf Khouloud**
Parallel Session S7		
13:00 – 13:15	Title:	Approximation of solutions for random fractional equations involving mean square Caputo derivatives
	Presenter:	**Hafssa Yfrah**
13:15 – 13:30	Title:	A new approximate analytical solution of fractional order nonlinear wave-like equations with variable coefficients
	Presenter:	**Khalouta Ali**
13:30 – 13:45	Title:	Solvability for a class of nonlinear fractional relaxation differential equations
	Presenter:	**Lachouri Adel**
13:45 – 14:00	Title:	Existence and Ulam stability of k-Generalized ψ-Hilfer Fractional Problem
	Presenter:	**Lazreg Jamal Eddine**
14:00 – 14:15	Title:	Abstract Nonlinear Boundary Implicit Caputo--Exponential Type Fractional Differential Equations
	Presenter:	**Malti Ahmed Ilyes Nedjib**
14:15 –	Title:	Existence and uniqueness of solutions for system of time-invariant fractional differential equations

14:30	Presenter:	**Mansouri Ikram**
14:30 – 14:45	Title:	A modified wright function for certain generalized fractional operators
	Presenter:	**Soumia Bourchi**
14:45 – 15:00	Title:	A discrete fractional covid-19 model existence and stability results
	Presenter:	**Noureddine Djenina**
Parallel Session S8		
13:00 – 13:15	Title:	Exact Asymptotic Errors and Bandwidth Selection for M-estimation under Truncated-Censored and Dependent Data
	Presenter:	**Hassiba Benseradj**
13:15 – 13:30	Title:	Semi-recursive kernel conditional density estimators under random censorship and dependent data
	Presenter:	**Sihem Semmar**
13:30 – 13:45	Title:	Strong consistency of a conditional mode estimator in the presence of doubly censored data
	Presenter:	**Hadjer Benchoulak**
13:45 – 14:00	Title:	Uniform convergence of nonparametric conditional hazard function in the single functional modeling for dependent data
	Presenter:	**Torkia Merouan**
14:00 – 14:15	Title:	Variable Selection strategy for zero inflated models with application to automobile insurance data.
	Presenter:	**Soumia Kaci**
14:15 – 14:30	Title:	Comparative study between two versions of Metropolis-Hasting algorithm for generating computer experiment designs according to a point process
	Presenter:	**Hichem Elmossaoui**
14:30 – 14:45	Title:	Adaptive gamma-BSPE kernel density estimatimation for nonnegative heavy tailed data
	Presenter:	**Yasmina Ziane**
14:45 – 15:00	Title:	Value-at-risk prediction using garch model and bayesian extreme value for mixture distributions
	Presenter:	**Redhouane Frihi**
Parallel Session S9		
15:30 – 15:45	Title:	On certain commutator estimates for vector fields on variable triebel-lizorkin spaces
	Presenter:	**Ben Mahmoud Salah**
15:45 – 16:00	Title:	On the Lp boundedness of a class of semiclassical Fourier integral operators
	Presenter:	**Elong Ouissam**
16:00 – 16:15	Title:	Multiple nontrivial solutions for a class of nonlinear elliptic Kirchhoff equations
	Presenter:	**Hayat Benchira**
16:15 – 16:30	Title:	Exponential stability for a delayed flexible structure with temperature and microtemperature effects
	Presenter:	**Houasni Mohamed**
16:30 –	Title:	Multiple solutions for nonhomogeneous elliptic equartions involving critical Caffarlli-Kohn-Nirenberg exponent

16:45	Presenter:	**Keddar Naima**
16:45 – 17:00	Title:	Vanishing viscosity for the navier-stokes Boussinesq system
	Presenter:	**Maafa Youssouf**
17:00 – 17:15	Title:	Asymptotic behaviour of solutions of nonlocal elliptic problems
	Presenter:	**Zaouche Elmehdi**
17:15 – 17:30	Title:	Variable Besov-type spaces
	Presenter:	**Zeghad Zouheyr**
Parallel Session S10		
15:30 – 15:45	Title:	Some Asymptotic Properties of the Conditional Set-Indexed Empirical Process Based on Dependent Functional Data
	Presenter:	**Souddi Youssouf**
15:45 – 16:00	Title:	Martingale Methods For Analysing The Non-Markovian Multiserver Retrial Queues
	Presenter:	**Houria Oukid**
16:00 – 16:15	Title:	Stability bounds comparison in the (R,S,LN Q) inventory model.
	Presenter:	**Nedjma Aiane**
16:15 – 16:30	Title:	A hybrid genetic algorithm for the protein structure prediction problem.
	Presenter:	**Nabil boumedine**
16:30 – 16:45	Title:	A new hierarchical secret sharing scheme
	Presenter:	**Meriem Ghanem**
16:45 – 17:00	Title:	Control of an Euler-Bernoulli beam with a nonlinear tension and an end-mass
	Presenter:	**Billal Lekdim**
17:00 – 17:15	Title:	Decomposition Based Parallel Hybrid MOEA with Application to the multiobjective multidimensional Knapsack Problem
	Presenter:	**Nedjmeddine Kantour**
17:15 – 17:30	Title:	A long-term study of the collective cell behavior of phytoplankton using the moment approximation method of an individual-based model (IBM).
	Presenter:	**Naziha Bordj**